



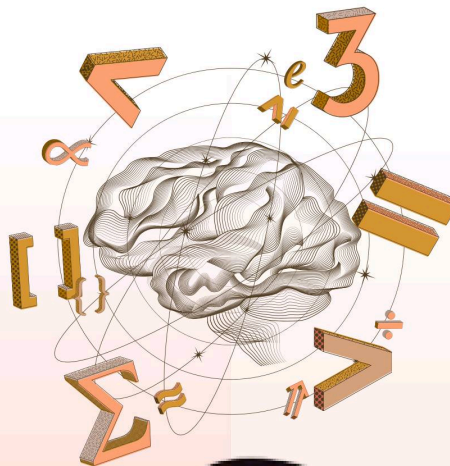
FROM *ENIGMA* TO *INSIGHT*: EXPLORING ABSTRACT ALGEBRA

The discipline of Abstract Algebra explores the profound abstractions of numbers and structures. It has evolved from its ancient origins into a powerful tool with numerous applications in modern science and industry.

The roots of abstract algebra can be traced back to early mathematicians such as Diophantus and Al-Khwarizmi. They solved equations and analyzed numerical systems. The development of algebraic notation by Francois Viète and the publication of Carl Friedrich Gauss' "Disquisitiones Arithmeticae" in the nineteenth century laid the foundations for modern algebra.

The significance of abstract algebra is evident in various fields. It is a fundamental concept in cryptography, coding theory, and computer science. Algebraic structures like groups, rings, and fields have practical applications in physics, engineering, and economics. Additionally, it plays a crucial role in understanding the symmetries of geometric objects and is integral to contemporary algebraic geometry.

Alan Turing, a pioneer in the field of computer science, made substantial advancements in abstract algebra through his research on Turing machines. His theories regarding computability and the halting problem are deeply rooted in the fields of mathematical logic and algebraic structures.



Evariste Galois, a highly esteemed scholar in the field of abstract algebra, is credited with the creation of group theory and the establishment of the foundational principles of Galois theory, which have been instrumental in solving polynomial equations. Emmy Noether made significant contributions to ring theory and formulated Noether's theorem, a fundamental principle in the realm of theoretical physics. The groundbreaking work of David Hilbert on finite fields, as well as his development of Hilbert's Nullstellensatz, has had a profound impact on the field of algebraic geometry.

Representation theory, as developed by esteemed mathematicians like Emil Artin and Richard Brauer, plays a crucial role in understanding the symmetries of algebraic structures. It is indispensable in comprehending group representations and their applications in fields such as quantum mechanics and particle physics.

Abstract algebra has undergone significant transformations throughout its development. The introduction of category theory by Samuel Eilenberg and Saunders Mac Lane marked a revolution in the field by providing a comprehensive framework for studying algebraic structures. Moreover, contemporary algebra explores concepts such as noncommutative rings and homological algebra, leading to further breakthroughs in various mathematical domains.

CONTENT BY: REEYA JOSHI



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Editors' note

When I entered college, I was immediately captivated by Algorithm's learning opportunities. I was drawn to its presentation style and the extensive research that diverged from the standard curriculum covered in class. Additionally, I was keen to establish connections with the senior members and get to know them better, and I'm pleased to say that I made a prudent choice.

I initially joined as a member of the Social Media Team in my first year, was promoted to the head of Social Media in the second, and now I stand in front of you all as the Editor-in-Chief for the session of 2023-24. Being part of Algorithm challenged the stereotype that mathematicians are solely studious and lacking creativity, as it encouraged us to explore our artistic and imaginative capabilities. The intricacies highlighted in our newsletter demonstrated the diverse and multifaceted nature of our team, which is truly remarkable.

Abstract algebra, often described as the "poetry of mathematics," is a discipline that unveils the hidden symmetries and patterns within mathematical structures. In this edition, we are delighted to present you with a selection of articles and essays that delve into various aspects of abstract algebra. From a beginner's guide to the fundamental concepts to explorations of its practical applications in our day-to-day lives, we aim to demystify and celebrate this fascinating subject.

As the Editor-in-Chief of this esteemed legacy within the Department of Mathematics at JMC, the team's unwavering dedication always motivates me to contribute at every single point, regardless of the circumstances. We have all put in dedicated efforts for this edition as well, and we aspire not to let you down. Enjoy reading!

Aditi Joshi
Editor-in-Chief

As students immersed in academia, knowledge is our lifeblood. My passion for working in an editorial environment dates back to my school days, and when I discovered that there was a dedicated Mathematics newsletter at JMC, I knew I had to be a part of it.

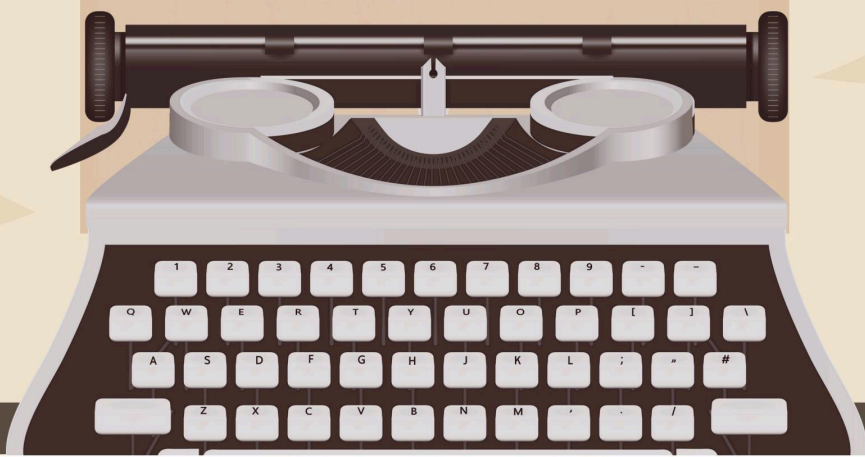
Aditi and I often talk about how Algorithm is such a cherishable part of the Math Department. JMC, as a liberal arts college, has kindled the artistic passions of many of us. However, what strikes me as remarkable is that, despite this creative environment, some of us hold mathematics so dearly that we invest significant effort in producing a dedicated newsletter for this field. In my opinion, that's truly commendable.

Mathematics, much like an exponential function, is constantly growing. It's the backbone of so much of what we have today. To illustrate, the words you're reading now owe their presence to abstract algebra. Without it, the internet, and countless software applications, including the one you're using to view this newsletter, wouldn't exist. It's fascinating how deeply mathematics is intertwined with everything we do. Whether pursuing arts or sciences, you inevitably encounter this discipline, and that's why I love it so much.

I'm immensely grateful for the opportunity to contribute to this newsletter. We've invested a lot of effort in creating this edition, and we hope you all like it.

Happy Reading!

Tanvi Sardana
Sub Editor-in-Chief



ORIGINS OF ABSTRACT ALGEBRA

Introduction

In mathematics, more specifically algebra, abstract algebra or modern algebra is the study of algebraic structures. Algebraic structures include groups, rings, fields, modules, vector spaces, lattices, and algebras over a field.

The term abstract algebra was coined in the early 20th century to distinguish it from older parts of algebra, and more specifically from elementary algebra, the use of variables to represent numbers in computation and reasoning. Presently, the term "abstract algebra" is typically used for naming courses in mathematical education, and is rarely used in advanced mathematics.

History

Before the nineteenth century, algebra was defined as the study of polynomials. Abstract algebra came into existence during the nineteenth century as more complex problems and solution methods developed. Concrete problems and examples came from number theory, geometry, analysis, and the solutions of algebraic equations. Most theories that are now recognized as parts of abstract algebra started as collections of disparate facts from various branches of mathematics and finally became unified in the early decades of the 20th century on a basis of a common set of concepts. This resulted in the formal axiomatic definitions of various algebraic structures such as groups, rings, and fields.

Elementary Algebra

The study of polynomial or algebraic equations has a long history. Around 1700 BC, the Babylonians were able to solve quadratic equations specified as word problems. This word problem stage is classified as rhetorical algebra and was the dominant approach up to the 16th century. Muhammad ibn Mūsā al Khwārizmī originated the word "algebra" in 830 AD, but his work was entirely rhetorical algebra. Fully symbolic algebra did not appear until François Viète's 1591 *New Algebra*, and even this had some spelled out words that were given symbols in Descartes's 1637 *La Géométrie*.

George Peacock's 1830 *Treatise of Algebra* was the first attempt to place algebra on a strictly symbolic basis. He distinguished a new symbolical algebra, distinct from the old arithmetical algebra. Whereas in arithmetic algebra $a-b$ is restricted to $a \geq b$, in symbolical algebra all rules of operations hold with no restrictions. Using this Peacock could show laws such as $-a-b=ab$, by letting $a=0$, $c=0$ in $a-bc-d=ac+bd-ad-bc$. Peacock used what he termed the principle of the permanence of equivalent forms to justify his argument, but his reasoning suffered from the problem of induction. For example, $a/b=ab$ holds for the nonnegative real numbers, but not for general complex numbers.

Early Group Theory

Several areas of mathematics led to the study of groups.

Lagrange's 1770 study of the solutions of the quintic equation led to the Galois group of a polynomial.

Gauss's 1801 study of Fermat's little theorem led to the ring of integers modulo n , the multiplicative group of integers modulo n , and the more general concepts of cyclic groups and abelian groups.

Klein's 1872 Erlangen program studied geometry and led to symmetry groups such as the Euclidean group and the group of projective transformations.

In 1874 Lie introduced the theory of Lie groups, aiming for "the Galois theory of differential equations".

In 1876 Poincaré and Klein introduced the group of Möbius transformations, and its subgroups such as the modular group and Fuchsian group, based on work on automorphic functions in analysis.

Arthur Cayley's 1854 paper *On the theory of groups* defined a group as a set with an associative composition operation and the identity 1, today called a monoid.

In 1870 Kronecker defined an abstract binary operation that was closed, commutative, associative, and had the left cancellation property $b \neq c \rightarrow a \cdot b \neq a \cdot c$, similar to the modern laws for a finite abelian group. Weber's 1882 definition of a group was a closed binary operation that was associative and had left and right cancellation.

Walther von Dyck in 1882 was the first to require inverse elements as part of the definition of a group.

Early Ring Theory

Noncommutative ring theory began with extensions of the complex numbers to hypercomplex numbers. In 1844, Hamilton presented biquaternions, Cayley introduced octonions, and Grassman introduced exterior algebras. James Cockle presented tessarines in 1848 and coquaternions in 1849. William Kingdon Clifford introduced split-biquaternions in 1873. In addition Cayley introduced group algebras over the real and complex numbers and square matrices.

In an 1870 monograph, Benjamin Peirce classified the more than 150 hypercomplex number systems of dimension below 6, and gave an explicit definition of an associative algebra. He defined nilpotent and idempotent elements and proved that any algebra contains one or the other. He also defined the Peirce decomposition.

Frobenius in 1878 and Charles Sanders Peirce in 1881 independently proved that the only finite-dimensional division algebras over $\mathbb{R}$ were the real numbers, the complex numbers, and the quaternions. In the 1880s Killing and Cartan showed that semisimple Lie algebras could be decomposed into simple ones, and classified all simple Lie algebras. Inspired by this, in the 1890s Cartan, Frobenius, and Molien proved (independently) that a finite-dimensional associative algebra over $\mathbb{R}$ or $\mathbb{C}$ uniquely decomposes into the direct sums of a nilpotent algebra and a semisimple algebra that is the product of some number of simple algebras, square matrices over division algebras.

For commutative rings, several areas together led to commutative ring theory. In two papers in 1828 and 1832, Gauss formulated the Gaussian integers and showed that they form a unique factorization domain (UFD) and proved the biquadratic reciprocity law. Jacobi and Eisenstein at around the same time proved a cubic reciprocity law for the Eisenstein integers. The study of Fermat's last theorem led to the algebraic integers.

In 1846 and 1847 Kummer introduced ideal numbers and proved unique factorization into ideal primes for cyclotomic fields. Dedekind extended this in 1871 to show that every nonzero ideal in the domain of integers of an algebraic number field is a unique product of prime ideals, a precursor of the theory of Dedekind domains. Overall, Dedekind's work created the subject of algebraic number theory.

In the 1850s, Riemann introduced the fundamental concept of a Riemann surface. His methods relied on an assumption he called Dirichlet's principle, which in 1870 was questioned by Weierstrass. Much later, in 1900, Hilbert justified Riemann's approach by developing the direct method in the calculus of variations. In 1882 Dedekind and Weber, in analogy with Dedekind's earlier work on algebraic number theory, created a theory of algebraic function fields which allowed the first rigorous definition of a Riemann surface and a rigorous proof of the Riemann-Roch theorem. Lasker in 1905, proved that every ideal in a polynomial ring is a finite intersection of primary ideals. Macauley in 1913, proved the uniqueness of this decomposition. Overall, this work led to the development of algebraic geometry.

In 1801 Gauss introduced binary quadratic forms over the integers and defined their equivalence. He further defined the discriminant of these forms, which is an invariant of a binary form.

In 1868 Gordan proved that the graded algebra of invariants of a binary form over the complex numbers was finitely generated, i.e., has a basis. Hilbert wrote a thesis on invariants in 1885 and in 1890 showed that any form of any degree or number of variables has a basis. He extended this further in 1890 to Hilbert's basis theorem.

The first axiomatic definition was given by Abraham Fraenkel in 1914. His definition was mainly the standard axioms: a set with two operations addition, which forms a group (not necessarily commutative), and multiplication, which is associative, distributes over addition, and has an identity element. In addition, he had two axioms on "regular elements" inspired by work on the p -adic numbers, which excluded now-common rings such as the ring of integers.

In 1920, Emmy Noether, in collaboration with W. Schmeidler, published a paper about the theory of ideals in which they defined left and right ideals in a ring. These definitions marked the birth of abstract ring theory.

Early Field Theory

In 1801 Gauss introduced the integers mod p , where p is a prime number.

Galois extended this in 1830 to finite fields with p^n elements.

In 1881 Leopold Kronecker defined what he called a domain of rationality, which is a field of rational fractions in modern terms.

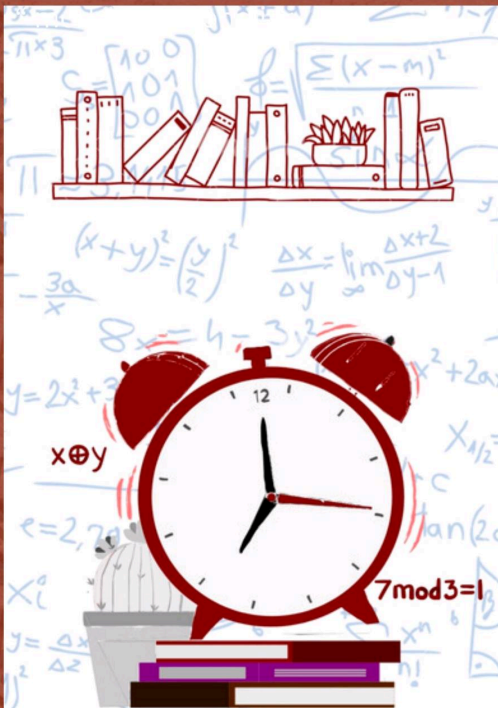
The first clear definition of an abstract field was due to Heinrich Martin Weber in 1893. It was missing the associative law for multiplication, but covered finite fields and the fields of algebraic number theory and algebraic geometry.

In 1910 Steinitz synthesized the knowledge of abstract field theory accumulated so far. He axiomatically defined fields with the modern definition, classified them by their characteristic, and proved many theorems commonly seen today.

MODULAR ARITHMETIC

In mathematics, modular arithmetic is a system of arithmetic for integers, where numbers "wrap around" when reaching a certain value, called the modulus. The modern approach to modular arithmetic was developed by Carl Friedrich Gauss in his book *Arithmeticae*, published in 1801.

Modular arithmetic is the branch of arithmetic mathematics related with the



Basically modular arithmetic is related with computations of "mod" of expressions. Expressions may have digits and computational symbols of +, -, *, / or any other.

Example: $17 = 8 \pmod{9}$

In the above example, 17 is dividend, mod 9 is the divisor and 8 is the remainder.

Parity Check Bits

Digital information is represented by bit string, split into blocks of a specified size. Before each block is stored or transmitted, an extra bit, called a parity check bit, is appended to each block. The parity check bit $x[n+1]$ for the bit string $x[1]x[2]...x[n]$ is defined

$$\text{by " } x[n+1] = x[1] + x[2] + \dots + x[n] \pmod{2} \text{ "}$$

→ It follows that $x[n+1]$ is 0 if there are an even number of 1 bits in the block of n bits and it is 1 if there are an odd number of 1 bits in the block of n bits. When we examine a string that includes a parity check bit, we know that there is an error in it if the parity check bit is wrong. However, when the parity check bit is correct, there still may be an error. This is used as an additional check to ensure that the packets of data received are correct and complete by the machine.

ISBNs

All books are identified by an International Standard Book Number, a 10-digit code. A new system of 13-digit ISBN code has recently been introduced to identify a large number of books. But here our main focus would be on ISBN-10. An ISBN-10 consists of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally the last digit is either a digit or the letter X (used to represent 10 in the Roman system). This check digit is

selected so that " $\sum i \cdot x[i] \equiv 0 \pmod{11}$, where $1 \leq i \leq 10$ ".

→ The above expressions can be simplified as " $x[10] = 1 \cdot x[1] + 2 \cdot x[2] + 3 \cdot x[3] + 4 \cdot x[4] + 5 \cdot x[5] + 6 \cdot x[6] + 7 \cdot x[7] + 8 \cdot x[8] + 9 \cdot x[9] \pmod{11}$ ".

The last digit also called the check digit is used to verify if there is any printing mistake in the ISBN. This is actually very interesting and you can verify yourself through the above simple calculation. In the new system ISBN-13, instead of 10, 13 digits are used and the check digit is always a digit, i.e. 0-9.

Modular Theory has many more applications to discrete mathematics, computer science, and many other disciplines; like generation of pseudo-random numbers, Hashing functions, Cryptography and even in Music!

12-Hour Clock

Modular arithmetic plays a vital role in understanding and managing cyclical events. If the length of the cycle is n , we refer to it as modulo n .

Consider the 12-hour clock, a common timekeeping device. It exemplifies the practical use of modular arithmetic. However, it is not merely a numerical device. Upon examination of its inner workings, one can discern an abstract algebraic foundation, specifically rooted in modular theory, which is a highly applied branch of mathematics.

Suppose, the clock currently reads 4 o'clock. What will it read after 4 hours? To arrive at the answer, we simply add $4+4$ which yields 8, indicating it will read 8 o'clock.

What time will it display after 12 hours?

After 12 hours, it will be 4 o'clock since the clock cycles back to the same time every 12 hours. If we use the previous addition method, we get the sum of 16 ($4+12=16$).

But 16 isn't on the clock. How do we account for this inconsistency?

Here comes the modular theory. The clock displays numbers ranging from 1 to 12. When the value exceeds 12, it "wraps around" back to 1 and continues incrementing. For instance, 13 o'clock becomes 1 o'clock, 14 o'clock becomes 2 o'clock and so on.

We write this mathematically as:

$$16 = 4 \pmod{12}$$

It signifies that, upon dividing 16 by 12, we get 4 as a remainder. The operator "mod" conveys that the clock cycles every 12 hours.



THE RUBIK'S CUBE



The Rubik's Cube is a well known puzzle that has remarkable group theory properties. The article is to understand how the Rubik's Cube operates as a group and explicitly construct the Rubik's Cube Group.

First, let's have a brief discussion about what a Group is?

So, a Group is a set of finite or infinite elements along with a binary operation (also known as group operation) which satisfies the four fundamental properties of Closure, Associativity, Identity element and Inverse element.

Rubik's cube has 43 quintillion possible configurations. Some basic rotations(moves) of the cube are: Right(R), Left(L), Up(U), Down(D), Front(F) and Back(B).

FOCUSING BACK ON THE RUBIK'S CUBE, HOW THE SET OF MOVES OF THE CUBE IS A GROUP DENOTED AS $(G, *)$?

- IN ORDER TO CALL $(G, *)$ AS A GROUP, ABOVE FUNDAMENTAL PROPERTIES SHOULD HOLD.
- LET $A, B \in G$, THEN $A * B \in G$. THUS, G IS CLOSED UNDER $*$.
- LET $M \in G$ AND 'E' BE AN EMPTY MOVE. WE APPLY THE OPERATION BETWEEN THESE TWO MOVES, $M * E = M$, I.E, G HAS AN IDENTITY ELEMENT.
- LET M BE AN ARBITRARY MOVE IN G , THEN M' IS THE REVERSAL OF STEPS OF MOVE M AND APPLYING GROUP OPERATION, WE GET THE IDENTITY ELEMENT, I.E,
 $M * M' = E$
 M' IS THE INVERSE OF M

THEREFORE, RUBIK'S CUBE FORMS A GROUP.

SYMMETRIES OF A RUBIK'S CUBE

The cube comprises 26 coloured blocks known as "cubies". There are 8 corner cubies and 12 edge cubies. Corner cubies have 3 visible faces, while edge cubies have only 2. The cubies with a single visible face are called centre cubies, and there are 6 centre cubies.

Rubik's cube has an immense number of possible configurations.

- Since the Cube has 8 corner cubies, possible positions of corner cubies are :

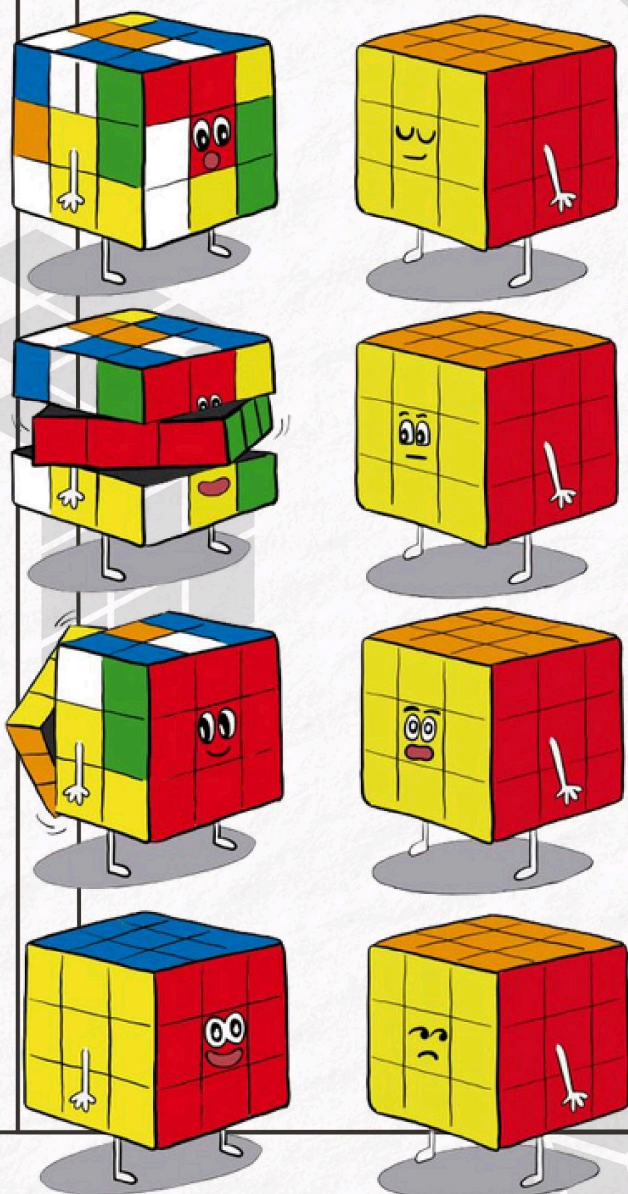
$$8! \cdot 3! \cdot 2! \cdot 1! = 8!$$

And since each corner cubie has 3 visible faces, the possible orientations are 38.

- Similarly, for edge cubies, total possible positions are 12! and total possible orientations are 212 since each edge cubie has 2 visible faces.

Thus, if we combine all of the different possibilities we see that $212 \cdot 8! \cdot 12!$ or 5.19×10^{20} potential configurations the Rubik's cube could be in- that is about 519 quintillion. However, it is important to note that some of these are actually not valid based on the starting configuration of the cube.

The transformations and configurations of the Rubik's cube form a subgroup of the group $(G, *)$, which is generated by the different horizontal and vertical rotations of the puzzle. The cube's position can be described as group elements, and the moves that can be made can be described as group operations. Understanding the cube's group structure allows for determining the minimum number of moves needed to solve the cube. Group theory is used to describe the symmetries of the cube, making the Rubik's cube the most practical application of group theory.



INTERVIEW

Algorithm: Would you like to introduce yourself?

Prof. Parvin Sinclair: Well, let me start by saying I'm a lover of mathematics, to the extent that it matters to me how people feel about mathematics, and that's why I'm interested in Math education as well. Over the years, I've become known in both the world of education and mathematics.

Algorithm: We have to know, when did you fall in love with mathematics?

Prof. Parvin Sinclair: My love for mathematics dates back to my childhood. Ah, the good old days. Mathematics always felt like something that was perfectly logical and made sense to me. I found it enjoyable and intriguing, and always leaned towards the abstract thinking aspect of it - although, even arithmetic involves abstract thinking - it's the first level of abstraction that most people are comfortable with. I was drawn to complex problem solving and problem posing, which is essentially what algebra is all about. The logical step by step movement that helped reach a conclusion - it captivated me from an early age. I don't remember not being fond of it.

Algorithm: Could you take us through your educational history and accomplishments?

Prof. Parvin Sinclair: My school years included 11 years of study, covering ninth, tenth, and eleventh grade together. My parents were both doctors and tried to persuade me to pursue medicine, but I was more inclined towards mathematics and economics. Fortunately, I had the option to study Math and Economics alongside other sciences. I enjoyed the mathematical aspects of economics, particularly the modelling. I then moved to Lady Shri Ram College, moved to IIT further and enjoyed myself thoroughly there in the Math Department. I later joined Tata Institute of Fundamental Research in Bombay as a Research Assistant for some time. However, I had to return to Delhi due to personal reasons, and that was when I joined IGNOU. At that time, IGNOU was just starting, but as one got into it it was wonderful to witness the endless opportunities, the potential of networking and connecting across the country, which was exciting.

Algorithm: What led you to the decision to start teaching abstract algebra?

Prof. Parvin Sinclair: What we call abstract algebra, is really what is algebra nowadays. Classical algebra, as it was historically known, primarily dealt with polynomials and their roots - some mathematicians concentrated on diophantine equations, Lagrange contributed significantly to this as well - whose works you can find in books like ET Bell's 'Men of Mathematics' and Courant and Robbins' 'What is Mathematics?'. They explore the concepts of roots and their relationships, how to solve for them and ultimately obtain them. However, in the late '80s and early '90s, mathematical thinking expanded. Mathematicians realised that when you zoom out and look at the broader picture, you encounter various structures and their properties. This marked the shift toward what we now study as algebra, particularly abstract algebra. This approach is axiomatic, logical, and helps us systematically analyse various mathematical structures.

Algebra acts as the foundational language and thought process for mathematics, including fields like analysis. It's important to note that classical geometry played a role in the evolution of abstract algebra, as formal algebra brought forth new possibilities. This transition opened up the world of algebraic geometry and geometric algebra, two different ways of exploring the connection between algebra and geometry. These qualities of algebra, its logical foundation, and its application in various mathematical areas have drawn me to the field. It's the backbone of mathematics, which easily attracted me to it, to delve deep and contribute to the discipline.

Algorithm: It can be challenging to shift from the way we typically learn mathematics as students to the realm of abstract algebra. Could you recount a particularly challenging abstract algebra problem or concept that you encountered during your research or teaching, something that intrigued you or posed a difficulty?

Prof. Parvin Sinclair: Algebra can be both exciting and challenging. For instance, consider Fermat's Little Theorem. Today, we often solve it as an exercise. The beauty of it lies in using the Euler's function to find the roots. Just by applying these concepts, you can prove a theorem in two minutes that was once considered difficult. This is how Gauss brought new insights to the problem. However, there have been numerous challenges throughout my work. My primary area of research was quadratic form theory and Witt theory. I happened to prove some things about Hermitian quadratic forms that others found challenging, results they hadn't considered proving. It's important to mention that I didn't work in isolation; I had many discussions with colleagues. These conversations often clarified my thinking and helped me explore new avenues. In research and teaching, there are always problems waiting to be solved, and collaboration with others is often the key to finding solutions.

Algorithm: What is your favourite theorem?

Prof. Parvin Sinclair: You know, it's hard to pick just one theorem. It really depends on the context. Euler's theorem is quite interesting, and I find the fixed point theorem quite fascinating, but there's no single favourite. It's not about the utility of the theorem for me; it's more about seeing how one came to it - that's what makes it a work of art. For example, when Cayley said every group is essentially a permutation group, it opened up exciting possibilities, even for solving equations.

So, these theorems each have their unique charm. They make up the rich tapestry of abstract algebra.

Algorithm: Considering the abstract and theoretical nature of abstract algebra, how do you make it accessible to your students?

Prof. Parvin Sinclair: Teaching abstract algebra depends on the learners and their background. For doctoral students with a strong mathematical foundation, I challenge them with advanced concepts and questions.

If I'm teaching at your level, I would consider your background and may introduce group theory by emphasising relationships and axioms. I point out that we use axioms in our daily lives, like the axiom of behaving decently. These concepts help bridge the gap between abstract algebra and real-life experiences. I believe in using specific examples from a learner's background to make the subject more relatable.

I don't strictly adhere to a teaching plan; instead, I focus on how the learners are reacting and adapting





my approach accordingly. Flexibility is essential in teaching, and understanding the dynamics of the group is key. If a plan isn't working, I adjust based on the learners' feedback and interaction with the material.

Algorithm: Are there any specific resources, books, or textbooks you'd recommend for students seeking a deeper understanding of abstract algebra?

Prof. Parvin Sinclair: For starters, it's great to see what's available in your library, and I'd encourage exploring various resources, like online platforms often provide articles, and newsletters that pose challenges and stimulate your thinking. Understanding the history of algebra is intriguing and offers insights into its development and philosophy. Though gender biases have contributed to the underrepresentation of women in science, remarkable figures like Emmy Noether, celebrated as a pioneer in algebra and ring theory, highlight the significant contributions women have made to these fields. Her foundational work laid the groundwork for the axiomatic approach to rings. She began by exploring integers and polynomials over integers, then expanded into polynomials over fields, exploring their properties. Dedekind was also instrumental in this evolution.

When I was in TIFR, I met Professor Parimala Raman who not only is an excellent algebraist, but she also gave the Noether Lecture in 2013 - a distinguished lecture series that honours women who have made fundamental and sustained contributions to the mathematical sciences. Olga Tausky's work in linear algebra, especially in Hermitian forms, is also worth exploring. These examples show that women have made important contributions to algebra and serve as inspirations for those who may have reservations about entering the field due to societal expectations. Providing such examples can help alleviate common fears.

I would also recommend exploring mathematics magazines like the one put out by the Math Association of America. Online subscriptions can provide access to a wealth of information. Kleiner is known for his engaging articles on various aspects of algebra. In India, Professor Passi tried to write about group theory and its development, but Kleiner's book is often more accessible for self-study. I'm a strong advocate of self-learning, so picking up a problem that challenges but doesn't overwhelm is a great approach. It's important to strike a balance.

For those interested in the history of algebra and its applications, you can delve into how algebra generalised from numbers to more abstract concepts, like Boolean algebra. Understanding the practical applications of algebra can be an exciting starting point. I recall that Professor Jain and Saroj Malik authored a book on the applications of algebra; you may find it in your library. Algebra plays a pivotal role in the development of technology, including the internet. Without algebra, many technological advancements would have never taken off. It's an exciting field with endless possibilities for exploration and understanding.

Algorithm: Could you share your insights on the evolution of the field of abstract algebra? Have you noticed any recent advancements, emerging trends, or practical applications of algebra in the real world that you'd like to discuss?

Prof. Parvin Sinclair: It's fascinating to see how computer-oriented applications and coding theory have gained prominence. There's a concept called "motives" in algebra that was initially developed by the Russian mathematician Voevodsky about a century ago. At the time, it didn't receive much recognition. But over the past 50 years or so, people have begun to appreciate how motives can be immensely valuable in revealing certain aspects in different areas of algebra.

In the past, from the 1960s to around the 1980s, algebraic topology experienced a decline. However, it started to reemerge with new questions and answers that intertwined with algebraic structures and topological spaces. These cycles of progress and decline are common in mathematical fields. Given the global landscape since the 1970s and 80s, much of the funding has been directed toward applications, technology, and science, and mathematics has seen increased emphasis on applications and technology-oriented research.

Fields have a wide range of applications, from polynomials to prime numbers, and these concepts are extensively used in cryptography. Linear algebra has applications in various fields, such as the study of problems like predator-prey and population dynamics. These problems involve linear algebra in their modelling and solution processes, often using differential equations.

Algorithm: How does one determine if they are a suitable candidate for a career in algebra?

Prof. Parvin Sinclair: It's not easy for someone else to definitively say if you're a suitable candidate for algebra. One thing to consider is your comfort with precision. Some individuals prefer a more flexible, less precise approach to problem-solving or enjoy more open-ended, creative tasks. While they might appreciate aspects of mathematics, algebra's precision may not align with their preferences. On the other hand, if you find satisfaction in solving problems and appreciating the process of arriving at a solution, algebra might be a good fit for you.

The best way to determine your affinity for algebra is to engage with it. Dive into algebra on your own, explore topics that interest you, and read materials at your own pace. School mathematics often doesn't do justice to the beauty and depth of algebra, and the fear of math can stem from teaching methods rather than the subject itself. Personal exploration and reading at your level can provide you with valuable insights.

A library can be a treasure trove, with knowledgeable librarians who can guide you. Eventually, you'll discover whether algebra resonates with you as you read and apply your mind to more complex concepts. Additionally, learning different languages is crucial since much of the original works in algebraic research is conducted in multiple languages, including French, German, and Russian.

Algorithm: If you weren't involved in algebra, are there any other mathematical fields that would have captured your interest?

Prof. Parvin Sinclair: I wouldn't have gone for applications - that's for certain. Honestly, I'd likely have delved deep into algebra exclusively - and it's important to note that algebra is an integral part of all mathematics. However, complex analysis is another area that is pretty attractive. The catch for me is that I can't tolerate hand-waving. (laughs) I crave immediate clarity, the ability to transform, document, and substantiate my understanding right there and then.

Algorithm: The distinction between abstract algebra and other branches of algebra, like elementary algebra and linear algebra, is often unclear to students. How would you explain the difference between abstract algebra and these other areas?

Prof. Parvin Sinclair: Elementary algebra encompasses the fundamental concepts, often introduced at the school level, such as working with variables and solving basic equations. On the other hand, abstract algebra extends these concepts. It ventures into deeper structures and properties of algebraic systems. Abstract algebra poses questions about the inherent properties of structures and explores their properties, which may or may not be similar to what's found in elementary algebra. As you're aware, Ramanujan primarily focused on Number Theory, using algebra as a tool to substantiate his mathematical concepts. It wasn't until his collaboration with Hardy that he embraced the notion of structured proofs and significantly increased his engagement in the proof-based approach.



Algorithm: What are some of the open questions in abstract algebra that researchers are currently exploring?

Prof. Parvin Sinclair: Open questions vary depending on the specific algebraic structures you're investigating. For instance, in arithmetic, you encounter intriguing questions such as when $A + B = A + C$, does it imply $B = C$? This holds in elementary arithmetic, but not necessarily for modules, which are certain structures over a ring. For modules A, B , and C , if you take A direct sum B and A direct sum C , B may not be isomorphic to C . Exploring the conditions under which this equivalence holds is a fertile area for research. There's so much work to do still, so much to prove! The field continues to evolve.

While it's true that funding in mathematics often leans towards applications and technology, organizations like the Clay Institute present opportunities for mathematicians. They offer substantial rewards for solving deceptively simple-looking but deeply complex mathematical problems. If you're interested, you can explore the challenges presented by the Clay Institute.

Algorithm: Lastly, would you like to share some profound insights or lessons you've gathered over your years of studying and teaching?

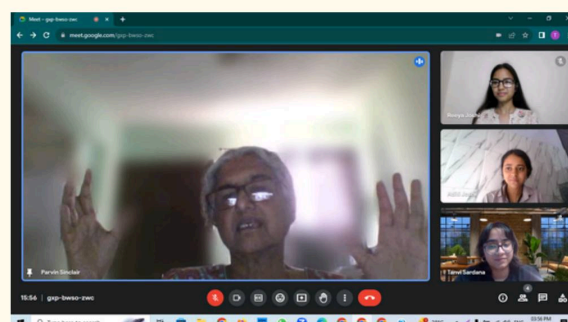
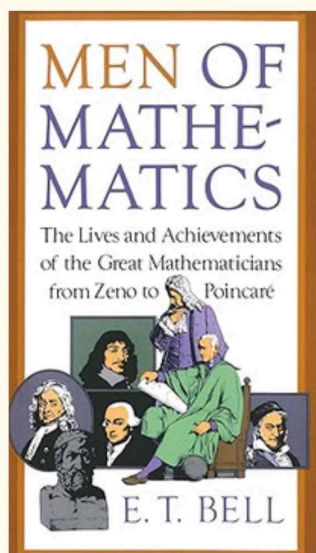
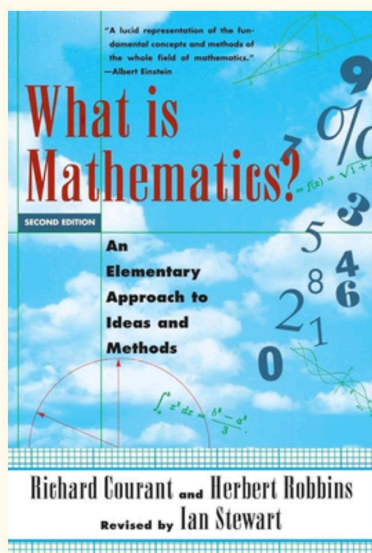
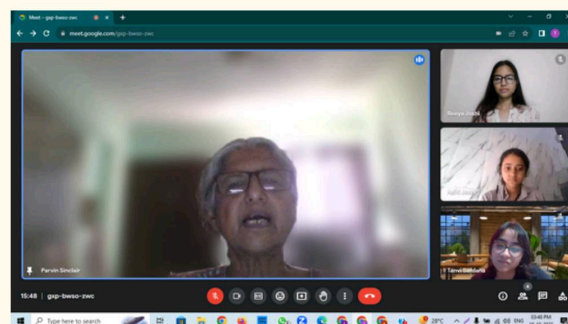
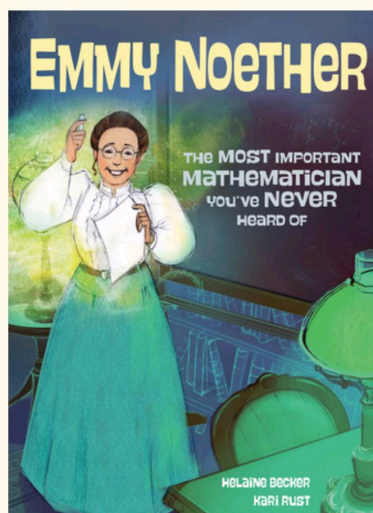
Prof. Parvin Sinclair: As an educator, one vital insight I've gained is to never assume you're finished with teaching. Sometimes, it may appear that students have grasped the ideas, but it turns out they may have different perspectives or questions that challenge your understanding. The key is not just solving the problem itself but finding the right way to explain it, ensuring both parties share a common conceptual understanding. Mathematics is characterized by simplicity and logic, but effectively communicating these concepts can be quite a challenge. Students often have to overcome societal perceptions that math is too difficult. It's not about making it easy; it's about making the effort worthwhile.

Additionally, if I could offer advice to my 20-year-old self as a student, I'd say this: don't expect miracles. Success is achieved through hard work, not innate genius. While some may label you as a genius, it's important to recognize that behind such recognition lies a wealth of dedicated effort.

Thank you for this insightful interview. If you ever come across questions or topics you'd like to delve deeper into, don't hesitate to reach out. I'm concerned about students relying too heavily on notes and not fully exploring the valuable resources available in libraries. Embrace the opportunity to explore, as libraries can ignite your passion. It was a pleasure conversing with all of you, and I hope you find inspiration in your mathematical pursuits.

EDITED BY : TANVI SARDANA

ILLUSTRATION BY: OJASVI KAPOOR & KAVNEET KAUR



MATHEMATICAL HEIGHTS

Mathematics forms a fundamental part of human thought and logic. It is not just about numerical values, theorems, and quadratic equations but much more. Mathematical exercise is an effective way of building mental discipline, logical reasoning, and mental rigor. Moreover, mathematical knowledge is also crucial to our understanding of other subjects such as natural science, social sciences, economics, statistics, computer science, information technology, biotechnology, aeronautics, and so on.

Taking up Mathematics in grad and post-grad binds the mindset of the students who are not afraid of numbers. Mathematics students think rationally, practically, and analytically and are more capable of handling the corporate world of digitalization. Mathematical literacy is taken to include basic computational skills, quantitative reasoning, spatial ability, and more!

They can Interpret data, charts, and diagrams, process information, solve problems, check answers, understand and explain solutions, and make decisions based on logical thinking and reasoning. Math permeates every aspect of our lives – at work and in practical everyday activities at home and beyond. For example, we employ mathematical functions when we go shopping, plan a holiday, decide on a loan, or decorate a room. It is not just students who need good numeracy skills. Parents also require it to help their children learn. Decisions in life are often based on numerical information. So, mathematical literacy is essential for making the best choices in life.

Many Universities like Delhi University, Jadavpur University, etc. admit students on the basis of marks in Class 12th. Hence, securing more than 80% marks in the 12th board is absolutely essential to secure a seat in a reputed university. A few Indian Institutes of Technology (IITs) also offer degrees in Mathematics or Mathematics and Computing. Admission to these programs is also through JEE Advanced.

Institute	Courses Offered	Entrance Exam
Indian Statistical Institutes Chennai Mathematical Institute Indian Institute of Science, Bangalore Indian Institutes of Science & Edu Research National Institute of Science & Education	B.Math/B.Statistics B.Sc Mathematics & Computer Science/B.Sc in Mathematics & Physics B.Sc Mathematics (Research) B.Sc & M.Sc dual degree in Mathematics Research Integrated M.Sc in Mathematics NEST	ISI Entrance Test, INMO B.Sc in Mathematics CMI Entrance Test JEE Advanced, JEE Mains KVPY JEE Mains, KVPY, 12th board NEST

Most careers in Mathematics require at least a Masters in Mathematics. Certain careers like University Professor, Economist, and Computer and Information Research Scientist require a doctoral degree (PhD). If you are sure that you want to eventually go for a career in Mathematics, pursuing an Integrated program saves you time and effort in preparing and applying separately for a master's degree. However, if you've completed a B.Sc in Mathematics, you can take JAM (Joint Admission Test for MSc.) for admission to Masters programs at IITs. Indian Statistical Institutes and Chennai Mathematical Institute also have their separate entrance exams for admission to M.Sc and Ph.D. programs. The top global universities for pursuing higher studies in Mathematics are:



- Oxford university
- Stanford university
- Harvard university
- University of California
- University of Toronto
- Massachusetts institute of technology
- University of Cambridge

Grad Record Examination (GRE):

The GRE General Test is frequently required if you're applying to graduate programs in mathematics or related fields, especially in the US. To evaluate your proficiency in mathematics, some programs may also request that you take the GRE Subject Test in Mathematics.

Grad Management Admission Test (GMAT):

Some programs in quantitative finance or business analytics may accept the GMAT even though it is primarily used for business school admissions.

Mathematics Subject Tests

(e.g., GRE Subject Test in Mathematics, SAT Math Level 2):

To evaluate your mathematical proficiency and readiness for advanced coursework, some mathematics programs or departments may demand or suggest that you take subject-specific standardized tests.

Careers in Mathematics

• Data Scientist

• Actuary

• Mathematician

• Statistician

• Astronomer

• Meteorologist

• Economist

• Physicist

• Engineer

• Market Researcher

• Mathematics or Science Professor

• Accountant/

Auditor

Mathematical Olympiads:

When applying to prestigious mathematics programs or research positions abroad, participation in international mathematics competitions like the International Mathematical Olympiad (IMO) can boost your resume.

Tests for Math Placement:

To determine which level of mathematics coursework is appropriate for incoming students, some universities may offer placement tests in the subject. When students arrive at the university, they frequently take these exams.

Exams for Graduate Admission

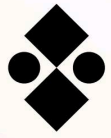
(country-specific):

For admission to graduate programs in mathematics or closely related fields, it may be necessary to take particular graduate admission tests or assessments, depending on the nation you are applying to. Check the prerequisites for the institutions you've selected.

In addition to these many mathematics graduates pursue research in government bodies like ISRO, DRDO, CSIR, etc. after completion of their post-grad.

Also many go to pursue a doctoral degree (PhD) from both Indian as well as renowned foreign universities and take teaching and research roles in university.

ALAN TURING & THE ART OF *Codebreaking*



Artificial General Intelligence (AGI):

General Problem-Solving:

AGI refers to AI systems that can comprehend, pick up knowledge, and use it across a variety of tasks and domains, much like a human.

Adaptability:

AGI is capable of adjusting to novel and unfamiliar circumstances, picking up knowledge from experience, and moving it from one domain to another. AGI is frequently thought to possess cognitive abilities that are similar to those of humans, such as reasoning, problem-solving, understanding natural language, and creativity.

Artificial Narrow Intelligence (ANI)

Narrow Focus:

ANI stands for artificial intelligence systems that are exceptionally good at carrying out particular tasks or solving particular issues within a narrow domain. They are intended for a specific, limited task.

Lack of Generalization:

ANI is unable to apply its knowledge or abilities to tasks other than those for which it was specifically trained. It needs a significant amount of reprogramming or retraining in order to adapt to new or unrelated tasks. Examples of ANI include image recognition software, recommendation algorithms, and virtual assistants like Siri or Alexa.

Artificial Super Intelligence (ASI):

ASI stands for artificial intelligence systems that are superior to humans in every way imaginable, including problem-solving, creativity, and abstract thought.

Unpredictable and Incomprehensible:

The capabilities of ASI may be so far beyond human comprehension. We might find its decision-making processes to be baffling.

Potential Risks and Benefits:

Because ASI's actions may have far-reaching effects on humanity, there are serious ethical and existential questions raised by its development. If not properly controlled, it could present unprecedented risks in addition to enormous benefits.

Why is Alan Turing's article still important today?

Alan Turing was a very famous person who was great at math, computer science, logic, and biology. He is known for creating the idea of theoretical computer science and explaining how algorithms and computations work.

He became famous for his important paper called "Computing Machinery and Intelligence", which was first published in 1950. Turing talked about a test called the Imitation Game, which he described as an experiment. The Imitation Game was a different version of the question "Can machines think?" To answer this question, understand what the words 'machines' and 'think' mean. Rather than trying to provide a specific explanation, change the question to a different one.

Turing believed that a machine could do any specific task if it was given the right instructions. He said that the universal Turing machine uses the same idea as the modern computer, which is that instructions and numbers are both just symbols.

The "Turing test" was a way to see if a computer could act like a human. In an experiment, someone would ask questions to find out if the answers were from a computer or a person. The concept is that if a machine can successfully pass the Turing test, then we can consider it as capable of thinking or being intelligent. To put it simply, up until 2021, no machine has been able to pass the Turing test. This shows that we still need to improve the "intelligence" part of artificial intelligence. Many people agree that the main objective of AI is to create artificial general intelligence (AGI) or artificial superintelligence (ASI). However, what we currently have is artificial narrow intelligence, which is mathematical models trying to understand and analyze data. But, there were several historical and cultural influences that shaped Alan Turing's work.

Tradition of Mathematics and Science:

Turing was born in 1912 in London, England, into a culture with a long history of mathematics and science inquiry. He had access to a wealth of mathematical information and resources because of the intellectual environment he was in.

World War II and Codebreaking:

The historical backdrop of the conflict had a significant impact on Turing's work on code breaking at Bletchley Park during World War II. His ground-breaking theories about computation were greatly influenced by the need to crack German encryption systems like the Enigma machine.

Social and legal influences:

One important cultural influence on Turing's life was his homosexuality. During his lifetime, homosexuality was a crime in the UK, and because of his sexual orientation, Turing ran into legal issues. Tragically, he underwent chemical castration, which eventually caused him to commit suicide in 1954.

Posthumous Recognition:

Turing's contributions to science and mathematics have gained more attention in the decades since he passed away.

One of the most esteemed honors in computer science is the Turing Award, which was first given out in 1966. Additionally, initiatives to acknowledge and express regret for the treatment he received unfairly because of his homosexuality gathered steam.

Legacy of computers:

The work of Turing laid the theoretical groundwork for contemporary computation and computers. Artificial intelligence (AI) and computer science have been greatly influenced by the Turing machine concept and Turing test.

Morphogenesis:

Later in his professional life, Turing developed a keen interest in biology, particularly the development of biological forms through the process of morphogenesis. His study of reaction-diffusion systems, which had an impact on the study of biological processes, was covered in his paper titled "The Chemical Basis of Morphogenesis."

These influences helped the readers of today's world better understand the significance of his contributions and the impact they had in the field of computer science.

Now we can say that we have prepared everything and are ready to start discussing our question: "Can machines think. " and the related question mentioned at the end of the previous section.

The Theological Objection is when people disagree with something based on their religious beliefs.

Thinking is something that our soul can do, and it never dies. Every person has an eternal soul given by God, but animals and machines do not have this. Therefore, no animal or machine is capable of thinking.

But this is just a guess. I don't think theological arguments are impressive, no matter what they are used for. These kinds of arguments have not been satisfying before.

The 'Heads in the Sand' Objection is when someone ignores or denies a problem or issue.

If machines were able to think, the outcomes would be really bad. Let's have hope and faith that they are unable to do it.

This argument is not often expressed as openly as described above.

We often think that humans are somehow better than everything else in the world. It's better if he can prove that he is better than others because then he won't risk losing his position of power.

I don't believe this argument is strong enough to need a counterargument. Comfort would be a better option: maybe we should look for solace in the idea that souls move from one body to another after death.

The Math Objection

Gödel's well-known theorem says that in any logical system that is powerful enough, there are statements that cannot be proven or disproven within the system unless the system itself is flawed.

It says that there are some things that this machine cannot do. If the imitation game is set up to answer questions, some of the questions will either be answered incorrectly or not answered at all, no matter how much time is given for a response.

Most people who believe in the importance of math would probably agree to use the imitation game as a starting point for discussion. People who agree with the two previous objections may not care about any standards or guidelines.

The core of the artificial intelligence (AI) field is the question of whether it is possible to build a machine that can actually think and be intelligent. This issue has been the focus of discussion and research for decades. Consider the following important details:



Mimicking Human Intelligence:

AGI does not imply that machines will necessarily think or be intelligent in the same ways that people do. Instead, it denotes the capacity for flexible and adaptable task performance and problem solving.

Current Progress:

As of my most recent knowledge update in September 2021, AGI has not yet been attained, despite significant progress in AI, especially in narrow domains (known as Artificial Narrow Intelligence, or ANI). While AI systems are excellent at some tasks, they lack human-level general intelligence and common sense.

Challenges and Complexity:

AGI development presents a number of complex challenges. It entails, among other things, comprehending and mimicking human cognitive processes, creating sophisticated machine learning algorithms, achieving natural language understanding, and dealing with ethical and safety issues.

The Argument from Consciousness is the idea that our consciousness is evidence of a higher power.

Professor Jefferson's speech in 1949 does a good job of explaining this argument. This argument seems to say that our test is not valid. According to the most extreme version of this belief, the only way to know for sure if a machine can think is to be the machine and experience the process of thinking. Then, you can tell others about these feelings, but nobody has to pay attention or think it's important. Similarly, according to this idea, the only way to understand someone's thoughts is to be that person. This is the solipsist perspective. This idea might make a lot of sense, but it can make it hard to share thoughts and understand each other.

Points of view from people with different disabilities.

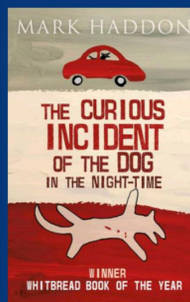
The idea that machines cannot make mistakes is surprising. Some people might answer, "Does it matter? " But let's try to understand their perspective with empathy. I believe this criticism can be explained using the example of the Imitation Game. It is said that the person asking questions could tell the difference between the machine and the man by giving them some math problems. The complaints we are discussing here are often hidden ways of arguing about consciousness.

Some other arguments were being made, such as the Argument from Informality of Behavior, the Argument from Continuity in the Nervous System, and Lady Lovelace's Objection.

Conclusion:

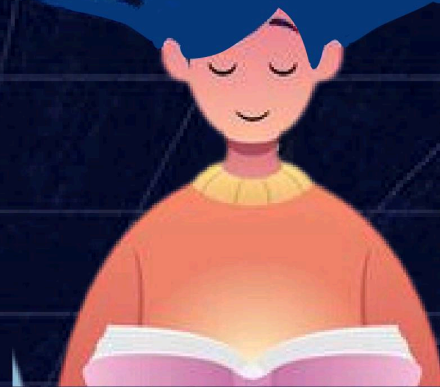
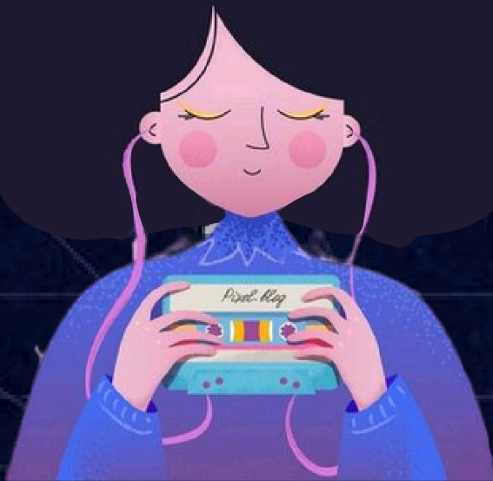
In our ever-evolving world of artificial intelligence, Alan Turing's pioneering ideas remain an essential touchstone. As we explore Turing's work, we uncover an intriguing journey. In today's world, many are focused on improving models with clever data techniques. But let's not lose sight of our grand goal. While our current methods are good, they're just a step. Figuring out artificial general intelligence (AGI) and artificial superintelligence (ASI) might not depend on what we know today. To reach our big dreams, we might need to find new ways. This can happen if we're open to new ideas and possibilities. Our thoughts and beliefs will guide us in making better methods for AI. Turing's legacy reminds us that creativity and imagination lights our way.

"Math Therapy," hosted by Vanessa Vakharia, is an enlightening podcast available on various platforms. It blends enjoyment and education to help overcome math trauma. With engaging guests, it demystifies mathematics, making it accessible and intriguing for all listeners, regardless of their math proficiency. A must-listen for conquering math fears.



Mark Haddon's "The Curious Incident of the Dog in the Night-Time" is a compelling mystery novel that delves into the mind of Christopher Boone, a young boy with autism. It unravels complex family dynamics and a murder mystery, offering a thought-provoking, emotionally resonant tale of resilience, love, and self-discovery. A must-read for those seeking unique and beautifully written narratives.

Darius Foroux's "Win Your Inner Battles" offers practical strategies to conquer procrastination and distractions in our fast-paced world. This invaluable book provides actionable advice for boosting productivity and achieving goals. For anyone seeking to navigate modern challenges and triumph over procrastination, it's a must-read guide for personal growth and success.



ALGORITHM RECOMMENDS



Good Will Hunting ADMIT 1 HALL 3

"Good Will Hunting," directed by Gus Van Sant, stars Matt Damon as Will Hunting, a janitor at MIT with exceptional mathematical talent and emotional struggles. The film, released in 1997, explores his journey of self-discovery, aided by therapy and a touching romantic relationship. It's a powerful story of love and intellect's transformative impact.



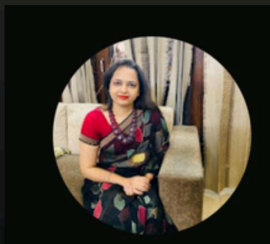
The Imitation Game ADMIT 1 HALL 2

"Imitation Game," starring Benedict Cumberbatch, is a must-see biographical drama based on "Alan Turing: The Enigma" by Andrew Hodges. It vividly portrays Alan Turing's life, his code-breaking brilliance during World War II, and the injustice he endured due to his sexuality. Cumberbatch's exceptional performance makes it a powerful, enlightening, and emotionally impactful cinematic journey.





MathTube



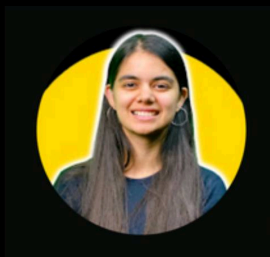
Maths Idol

"Maths Idol" on YouTube is a valuable resource for undergraduate-level mathematics students. Its extensive library simplifies complex topics, making maths more accessible. Whether you're learning or revising, this channel offers comprehensive classes, making it an essential destination for mastering undergraduate-level mathematics. Don't miss out on this helpful resource.



MathTV

If you're on a quest to sharpen your mathematical skills or simply find joy in the world of numbers, 'Math TV' is a YouTube channel that deserves your attention. This channel offers a diverse range of content, from basic arithmetic to advanced calculus and beyond. The channel instructor gives clear instructions making maths more fun than we imagined it could be. It is huge how interactive their sessions are with quizzes and problem solving exercises.



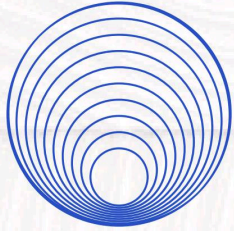
Apna College

"Apna College," hosted by Shradha Khapra on YouTube, is a superb channel for academic and career advancement. Shradha imparts valuable academic skills, interview tips, and insights on placements, internships, and productivity. Her inspiring leadership in the IT industry makes it a must-watch for students and professionals seeking personal and career growth.

CONTENT BY: PALAK SINDWANI

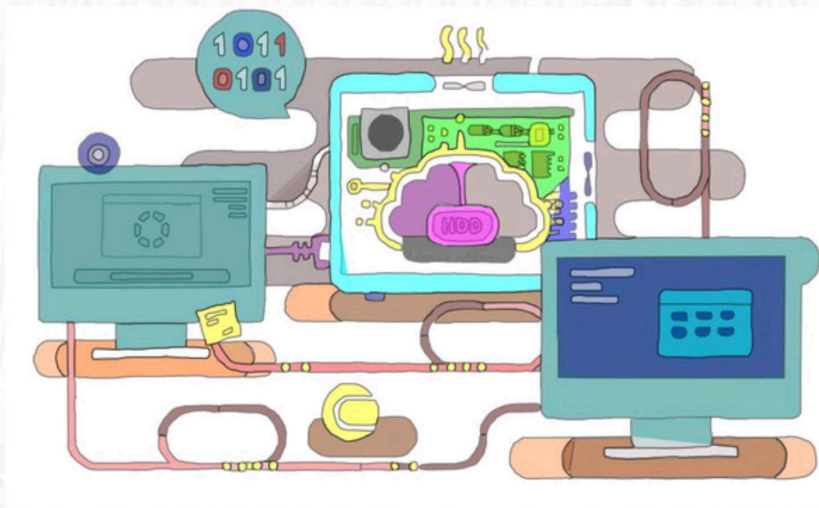
Use of Linear Algebra

in



ERROR CORRECTING CODES

If we wish to transfer a message to a destination in the form of bits there are chances that there may occur some errors which need to be detected. One such method we knew was Shannon's Noisy Coding Theorem which tells us, that if the redundancy is above a certain level (constrained by the noise of the channel),



then we can drive the probability of error (detecting the wrong message sent) to zero with increasing block length. The proof was based on a random code which is not very practical. For easy encoding and decoding, we need structure in our encoding/decoding functions.

Hamming Code(7,4)

Linearity allows an easier method of detecting errors called linear block codes and their structure offers several advantages. A linear error-correcting code known as Hamming(7,4) in coding theory converts four bits of data into seven bits by adding three parity bits. A single-bit error in a data bit or a parity bit can be found and fixed thanks to the Hamming codes, which aim to create a set of parity bits that overlap. Although many overlaps can be made, Hamming codes present the general technique. The Hamming code is a single error correction linear block code with $(n, k) = (2^m - 1, 2^m - 1 - m)$, where $m = n - k$ is the number of check bits in the codeword. The simplest non-trivial code is for $m = 3$, which is the (7, 4) Hamming code. The generator matrix of the Hamming code has dimension $k \times n$ and is of the form

$G = [k \times n]$. For this Hamming code, S has one row for each possible m -bit string with a weight of at least 2. There are exactly $k = 2^m - m - 1$ such strings. Any Hamming code is a 1-error correcting code as the two conditions above are satisfied. The (7, 4) Hamming code is given by the generating matrix: $G = [1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1]$. We know how to encode with any linear code. The codeword generated by message m is simply $c = mG$. But how about decoding?

If no errors were introduced, the first k bits constitute the message. But errors might have been introduced.

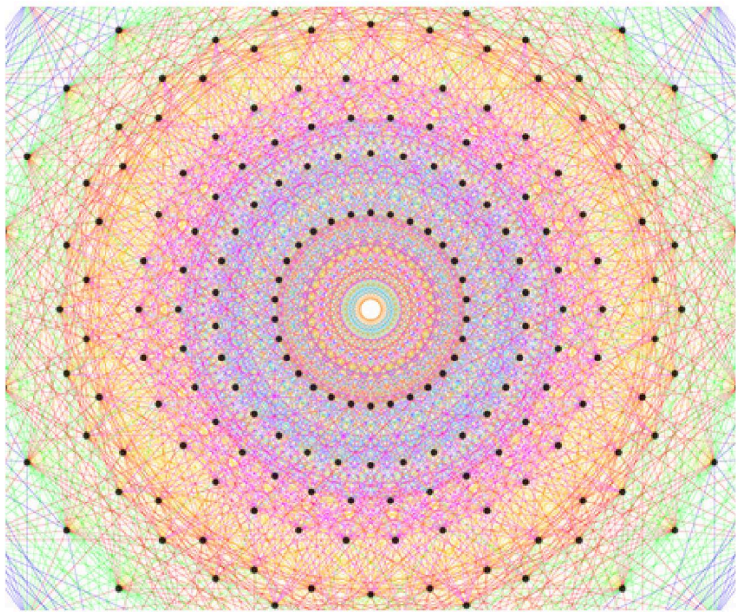
When there is only one error, decoding is easy as we show now.

A parity-check matrix H with the formula $H = [S]$

exists for each generator matrix $G = [I\ S]$. We know that the identity matrix is of size $(n-k) \times (n-k)$ because S has size $k \times (n-k)$, and H has size $n \times (n-k)$. The crucial finding is that $GH = S + S = 0$, and as a result, $c = mG$ satisfies the condition that $cH = mGH = 0$.

By computing the syndrome cH , which should be $cH + e_iH = e_iH$, we can identify the error e_i if there is one and we have received $c = c + e_i$. Thus, cH must be one of the rows of H and the index of the row indicates which bit has been corrupted. Once we know which bit was corrupted, we can recover the message.

THE *Representation Theory*



Representation theory is a powerful mathematical framework with broad applications in fields like physics, chemistry, computer science, and cryptography. It helps us understand and manipulate complex structures by representing abstract algebraic entities like groups and rings as linear transformations on vector spaces, making it a valuable tool for analyzing and solving problems related to symmetry.

Advancements in Representation Theory: In recent years, machine learning and data analysis have benefited from representation theory. Researchers have employed group representations to design algorithms for various tasks, such as image recognition, natural language processing, and recommendation systems. Representations allow for efficient data compression and extraction of essential features, improving the performance of machine learning models.

Algebraic Geometry: the theory has connections with algebraic geometry, a branch of mathematics that studies algebraic equations and geometric shapes, as they can often be represented by algebraic structures & representation theory provides a tool to study their symmetries.

Number Theory: Representation theory has found applications in number theory, particularly through its connection with modular forms and elliptic curves.

Topology and Knot Theory: Knot theory is another area where representation theory has made significant contributions. Through the use of braid group representations, researchers have discovered new knot invariants and provided insights into the classification of knots. This has implications for fields such as DNA research and material science.

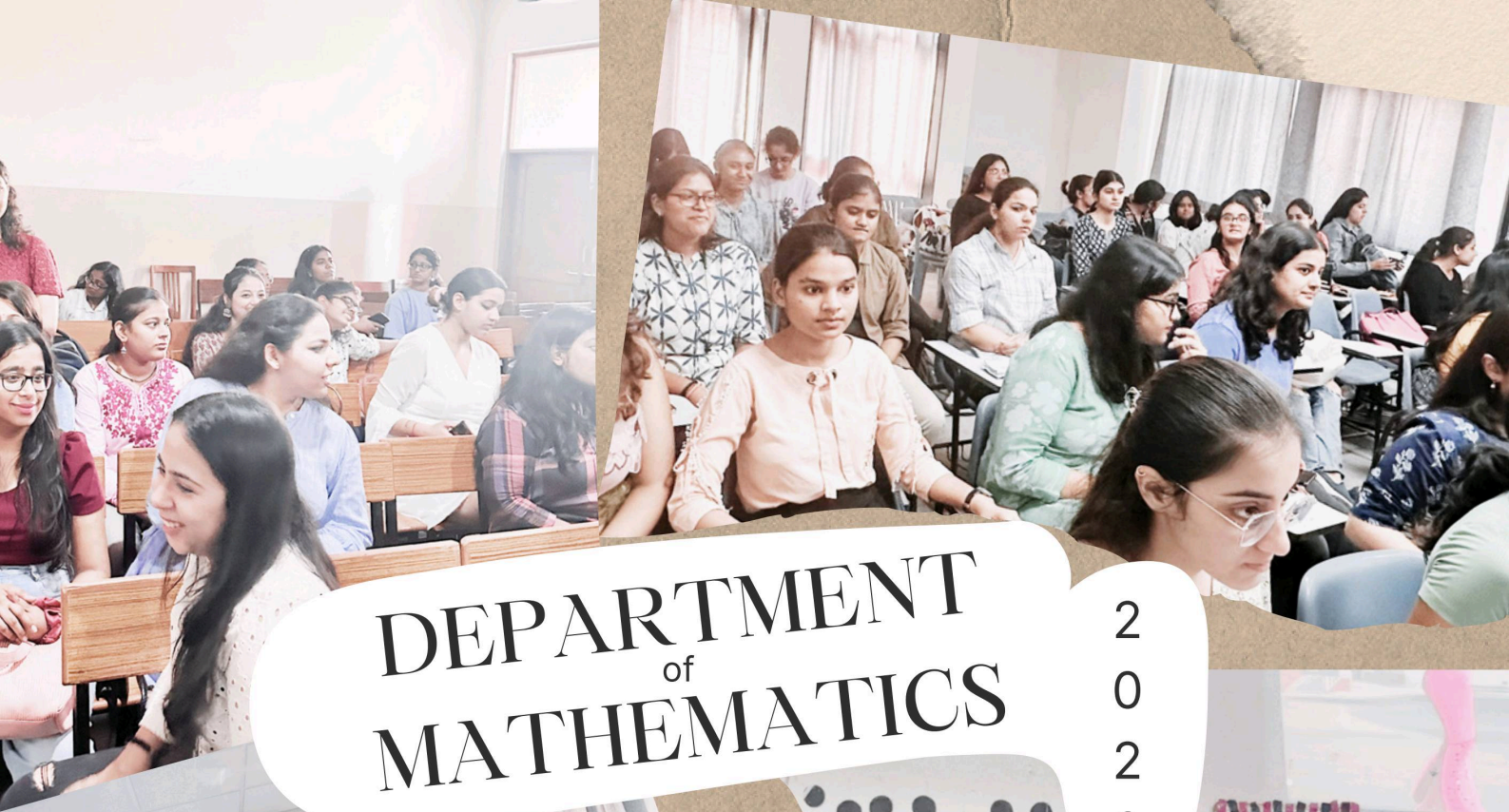
Quantum Mechanics: Physicists use group representations to describe the symmetries of physical systems and to classify elementary particles.

Why Representation Theory Matters

Representation theory isn't merely theoretical; it's a potent tool with broad applications in science and technology. From quantum systems to machine learning and abstract math, it offers a versatile framework. It also acts as a bridge between theory and real-world problem-solving, translating abstract math into practical solutions.

Conclusion

Advancements in representation theory have opened up exciting opportunities for researchers in physics, mathematics, computer science, and beyond.



DEPARTMENT of MATHEMATICS

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3



FAREWELL'23

On May 1st, 2023, the Department bid adieu to its final year students with smiling faces and heavy hearts. Everyone came together to present a splendid final show for them. The day started with a welcome address by Tahira Khan, the Vice President of the department. The second years kicked off the fun by performing an electrifying dance performance. Then, the amazing vocalists of the department made everyone teary eyed with a beautiful medley of songs, which was followed by another power packed dance by the first years. The third years, then, left everyone in awe as they showcased their talents and competed for the coveted titles of Miss Mathematics and Miss Farewell. They performed dances, sang melodious songs and believe it or not, even did pushups in a saree! The shortlisted students answered thought provoking questions after which three of them emerged as winners. Atipriya Dev Sinha was crowned Miss Mathematics, Jeffy Sabu secured the Miss Farewell title and Prashita Verma became Miss Personality. The seniors took a trip down the memory lane as the social media team presented a nostalgic montage video that brought them to tears. Making it even more difficult to hold back tears, Vashima, the Senior Advisor, delivered a beautiful parting speech. Devika Raaj Gupta, the President of the Department, delivered the Presidential Address and concluded the event on a heart touching note. The third years captured their memories in their cameras and danced together for the last time. It was truly the most memorable day for everyone.



TEACHER'S DAY

On the 5th of September 2023, the Department of Mathematics celebrated Teacher's Day with great enthusiasm and fervour. The event was a resounding success, with students from all years coming together to honour the tireless efforts of our mentors. The event began with a speech from the President of the department, Prakriti Raman. It was followed by some fascinating performances by the talented members of our department. The captivating dances left everyone smiling and tapping their feet. Then came the melodious song medley that enchanted the teachers and the students alike. The highlight of the event was The Dare Game. Our teachers were all given certain 'dares' which unveiled an unseen side of their personalities. They sang songs, mimicked actors, danced and even performed a ramp walk with the students! Our professors, then, came together for the cake cutting ceremony, marking the end of a truly memorable event. Later, students presented tokens of gratitude to the teachers. The event concluded with a Vote of Thanks from the Vice President, Vijay Bharti, highlighting the crucial role of teachers in shaping the future. It was a day filled with love and respect, honouring our professors' unwavering commitment.

FRESHERS' 23

On October 6, 2023, the Department of Mathematics warmly welcomed our lovely freshers with a vibrant Barbenheimer-themed fresher's party. The event took place on the third floor in classroom 408. The event began with an energetic entry of the freshers, dressed to the nines and grooving to foot tapping music, while the seniors turned paparazzi shone flashlights at them. The program officially commenced with welcoming remarks from the anchors, followed by an address from the Department President, Prakriti Raman. The event featured a flawless dance and a musical medley by senior students, along with snacks for everyone. The first-year students had their moment in the spotlight, when they walked the ramp and showed off their amazing outfits. The highlight of the day was the Miss Mathematics and Miss Personality competition, where participants entertained the audience with performances and displayed their wit in the questionnaire round. After another thrilling dance performance by the seniors, Devyaani Lakhota was crowned Miss Mathematics, and Shaivi Shubh won Miss Personality. The teachers felicitated the winners, and the event concluded with a vote of thanks from the department's vice president, Vijay Bharti. The day ended with a lively group photo and students dancing till their feet hurt, making it a highly successful and enjoyable occasion.



SPEAKER SESSION

On the 20th of October, 2023, Prof. (Dr.) Swarn Singh, an expert in Computational Methods for Differential Equations and Numerical Analysis and tenured professor at Sri Venkateshwara College, spoke at a session held by the Department of Mathematics. The event began with a felicitation of Prof. Singh and a warm welcome speech by Prakriti Raman, the President of the Department. The entire department enjoyed learning in depth about mathematical topics like The Battle Model, SIR Model, and The Exponential Decay Model. Prof. Singh made these topics relatable by providing real-life examples, like the violence in the Gaza Strip, to explain the Battle Model. During the session, Prof. Singh shared his personal journey and how he became interested in Differential Equations, emphasizing the connection between Real Analysis and Differential Equations. His words left the department fascinated and inspired. The event concluded with a Vote of Thanks from the Vice President, Vijay Bharti. Overall, the session unraveled the mysteries of Differential Equations and provided a fresh perspective on mathematical modeling.



ACHIEVEMENTS

ANAM (Batch of 27') - First Position in Numerical Narratives organised by Algorithm and Cauldron. Secured 3rd Position in EndGame Competition organised by Commerce Department.

RIDDHI SINGHAL (Batch of 27') - Served as a member of OC at Maiden edition of Alitor Delhi Model United Nations 2023 at Satyawati College.

PRAKRITI RAMAN (Batch of 24') - First Position in Inter Department Quiz Competition organised by Curiosus Society in JMC.

SHREYA (Batch of 24') - 5th position in RamanuGen- Mindtrix Quiz Competition at Ramanujan College.

ADITI JOSHI (Batch of 24') - As a part of Nrityanjali, the Indian Dance Society of JMC, got the first position in group folk dance competition in springfest organised by IIT Kharagpur.

SAI STUTI MITTAL (Batch of 24') - Lead Organiser of TEDxJMC.

DAMINI BAKHSHI (Batch of 24') - President of 180dc JMC.

MUSKAAN BABBAR (Batch of 24') - General Secretary of Training and Development Division, Placement Cell, JMC. Placed at PWC through Campus Recruitment.

KHYATI SHARMA (Batch of 24') - Vice President of Chanakya, The Economics Cell, JMC. Placed at PWC through Campus Recruitment.

TANVI SARDANA (Batch of 24') - President of Training and Development Division, Placement cell, JMC. Placed at Bain and Company through Campus Recruitment.

AARUSHI SONI (Batch of 24') - President of Puzzle Society, JMC.

RISHIKA AGGARWAL (Batch of 24') - First position in Mathematics Honours in First year.

SAACHI SAHNI (Batch of 24') - Received NCC-C certificate.

MANASVI MITRA (Batch of 24') - Senior Advisor at Digilit, JMC.

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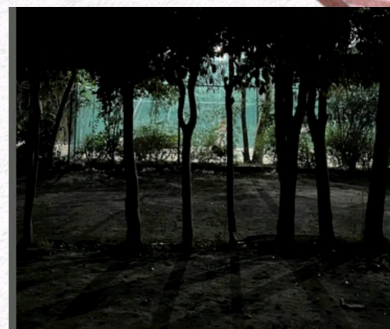
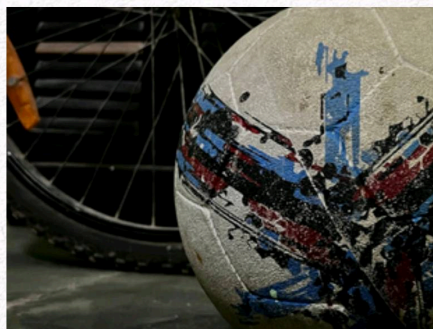
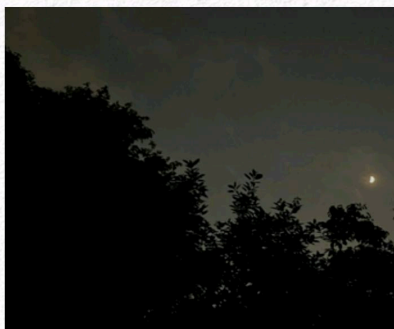
Ria Kapoor

NUMERICAL NARRATIVES

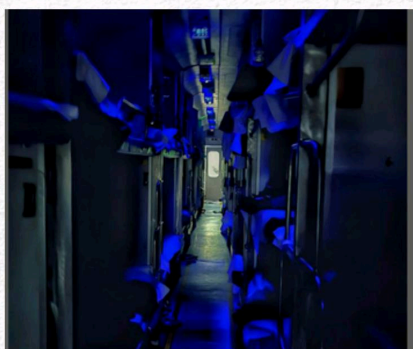
CAULDRON



ALGORITHM



On the occasion of **World Maths Storytelling Day** on 25th September, Cauldron, the Magazine Society of Jesus and Mary College and Algorithm, the newsletter of the Mathematics department organized **Numerical Narratives - A creative writing and photography competition for all JMC students.**



SANJANA DUTT

WINNER: PHOTO ESSAY

Math tells us three of the saddest love stories- Tangent lines who had one chance to meet and then parted forever; Parallel lines who were never meant to meet; Asymptotes who can get closer and closer but will never be together.

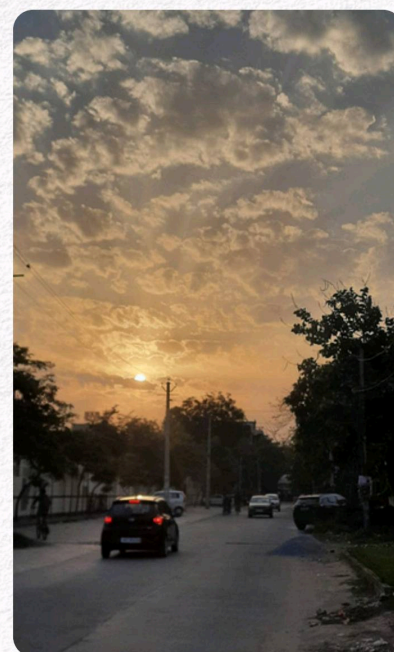


NAKSHATRA

FIRST RUNNER UP:

PHOTO ESSAY

Emotions can be perceived as either states or processes. It's a profound phenomena woven together by a multitude of human experiences. The photograph symbolically illustrates the complex interplay between the environment and human disposition. The people here respond uniquely to a sudden downpour; one's way of response reveals the true shades of their underlying state. 'Fractal is indeed the way of seeing infinity'.



KHUSHI GOYAL

HM: PHOTO ESSAY

At the edge, where heaven meets the land
Clouds, like artists, craft their strands
Jagged edges, like fractal lace
Intricate beauty in every place
So look above, to the canvas on high
Where fractals and clouds freely lie
In the heavens' gallery, open and free,
A masterpiece of wonder for all to see.



ANAM

WINNER OF SHORT STORY WRITING

Life isn't just a set of theorems that are fixed but life is a series of random events that are uncertain...

We walk with correct steps or we never get the answers."

In the heart of a bustling city, amidst the chaos of everyday life, lived Hayat. Her name itself meant 'life'. She was struggling with crippling insecurity and pervasive loneliness. She moved through life like a ghost, barely noticed by her peers, feeling like a misplaced puzzle piece in a vast, complex world.

One unexpected day, while wandering through a park, Hayat encountered a girl named Hazel. She exuded warmth and kindness, and her eyes held a glimmer of understanding that drew Hayat in. Hazel's gentle demeanor and genuine care made Hayat feel, seen for the first time.

"Life often unfolds through chance encounters,"

Hayat wondered as she sat by Hazel in the park, admiring the play of sunlight on the leaves. Hazel's presence brought a newfound warmth into Hayat's life, and they quickly became inseparable. Every day, they would meet and sit under the tree, talking about their fears, dreams, and life's uncertainties.

As their friendship blossomed, Hayat's parents noticed the positive change in her demeanor. However, they couldn't help but wonder about the origin of this newfound friend and were concerned about it.

Hazel became Hayat's confidante, her happiness and her peace. Hayat found solace in Hazel's presence, a connection so profound that she began to confide in her every day.

Hayat became more dependent on Hazel as all she ever wanted to do was to play or talk with her and that worried her parents. They decided to consult a therapist, hoping to ensure Hayat's emotional well-being.

Dr. Stephen, a compassionate therapist, patiently listened to Hayat's day to day life story, navigating the intricate web of Hayat's emotions and the emergence of Hazel in her life.

"Probabilities shape our experiences", Dr. Stephen gently explained to Hayat, broaching the possibility that Hazel might be a manifestation of her mind—a way to cope with her insecurities and loneliness. The revelation sent shivers down Hayat's spine, challenging her understanding of reality.

Weeks passed, and Hayat began to unravel the complex layers of her psyche with Dr. Stephen. She realized that understanding her own mind was a journey of odds and possibilities. With deeper understanding of her mind, she started recognizing the signs of Dissociative Identity Disorder (DID) within herself. "Life's dice had rolled, revealing an unexpected facet of my reality," Hayat whispered to herself, coming to terms with the existence of Hazel as an alter personality and cried her heart out.

Integration became Hayat's quest—a chance to unite her fragmented self and confront her insecurities head-on. Dr. Stephen guided her through the labyrinth of her emotions, helping her bridge the gap between reality and her imagined companion. Life's probability was at play, offering a path to healing for Hayat. In time, Hazel merged with Hayat, like the coming together of intersecting lines and she had to say goodbye to her friend, Hazel. But, Hayat emerged stronger, armed with the wisdom that almost every aspect of life hinges on probabilities—a roll of the dice that defines our existence.

Life continued, with Hayat now embracing the reality of her own mind. She realized that the love she had for Hazel was a reflection of the love she needed for herself. "Almost all of Life depends on Probabilities," she believed, cherishing the lessons learned through the complex dance of probabilities in her life.



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Algorithm, Jesus and
Mary College, 2023



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Algo team meet
25/10/2023



Algo team meet
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